How to know when you are being cheated or

Hypothesis Testing with χ^2

$$\chi^2 = \sum \frac{(obs - exp)^2}{exp}$$

Example: Heads vs. Tails

There are two categories of observations, H and T so there are two values to sum.

$$X^2 = \frac{(obsH - expH)^2}{expH} + \frac{(obsT - expT)}{expT}$$

Degrees of freedom (df = number of categories - 1 = 1)

(b)

df	Probability (p)					
	0.90	0.50	0.20	0.05	0.01	0.001
1	0.02	0.46	1.64	3.84	6.64	10.83
2	0.21	1.39	3.22	5.99	9.21	13.82
3	0.58	2.37	4.64	7.82	11.35	16.27
4	1.06	3.36	5.99	9.49	13.28	18.47
5	1.61	4.35	7.29	11.07	15.09	20.52
6	2.20	5.35	8.56	12.59	16.81	22.46
7	2.83	6.35	9.80	14.07	18.48	24.32
8	3.49	7.34	11.03	15.51	20.09	26.13
9	4.17	8.34	12.24	16.92	21.67	27.88
10	4.87	9.34	13.44	18.31	23.21	29.59
15	8.55	14.34	19.31	25.00	30.58	37.30
25	16.47	24.34	30.68	37.65	44.31	52.62
50	37.69	49.34	58.16	67.51	76.15	86.60

χ^2 values

Hypothesis Testing with χ^2

10 Tosses

Heads		Tails		χ^2	Probability
Obs	Exp.	Obs.	Exp.		**df = 1**
5	5	5	5	0	P > 0.9
4	5	6	5	0.4	0.5 < P < 0.9
3	5	7	5	1.6	0.2 < P < 0.5
2	5	8	5	3.6	0.05 < P < 0.2
1	5	9	5	6.4	P < 0.05*

100 Tosses

50	50	50	50	0	P > 0.9
45	50	55	50	1	0.2 < P < 0.5
40	50	60	50	4	P < 0.05*
30	50	70	50	16	P < 0.001***

(b)

df	Probability (p)					
	0.90	0.50	0.20	0.05	0.01	0.001
1	0.02	0.46	1.64	3.84	6.64	10.83
2	0.21	1.39	3.22	5.99	9.21	13.82
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χ^2 values

Testing of Mendel's Data

- Erect a model with a Null Hypothesis:
  - Laws of Unit Factors, Dominance, Segregation, and Independent Assortment hold.
- Predict values:
  - Do branch diagram to get expected values.
- Test with χ^2:

Parents Yellow Round **GGWW** x Green Wrinkled **ggww**

F1 **GgWw**

• F2	Obs	Expected	$((obs - exp)^2)/exp$
• **G_W_**	315	9/16 * 556 = 312.8	0.015
• **G_ww**	101	3/16 * 556 = 104.3	0.104
• **ggW_**	108	3/16 * 556 = 104.3	0.131
• **ggww**	32	1/16 * 556 = 34.8	0.225
• Total	556		**$\chi^2 = 0.475$ df = 3**

P > 0.9

» Fail to Reject (**ACCEPT**) hypothesis; with this data Mendel's postulates seem to be holding.

(b)

df	Probability (p) 0.90	0.50	0.20	0.05	0.01	0.001
1	0.02	0.46	1.64	3.84	6.64	10.83
2	0.21	1.39	3.22	5.99	9.21	13.82
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χ^2 values

If we ran this experiment many times, we would expect to see the results deviate from the expected this much or more 90% of the time… another words, almost all the time, so the model (Mendel's Laws) works pretty well.

What are the actual probabilities of various outcomes? Binomial Probabilities

$(n!/s!t!)\ a^s\ b^t$

n = total number of events
s = obs. # of outcome 1
(say heads or girls)
t = obs. # of outcome 2
(say tails or boys)
a = probability of outcome 1
b = probability of outcome 2

Example: **What is the probability of having 6 girls in a family of 6?**
$6!/6!*0!\ \ (0.5)^6 * (0.5)^0$
P = 1/64 = 0.016

Example: **What is the probability of having 4 girls in a family of 6?**
$6!/4!*2!\ \ (0.5)^4 * (0.5)^2$
P = 15/64 = 0.23

Formalizing some basic ideas about probability and multiple events.

Parents Yellow Round x Green Wrinkled

GGWW **ggww**

F1 **GgWw**

X

F2 9 **G_W_** : 3 **G_ww** : 3 **ggW_** : 1 **ggww**

Probability involving independent events:
What is the expected proportion of Green Round seeds?

Probability involving alternative events:
What is the expected proportion of seeds that are not like original parents?

Conditional Probabilities:
Of the Yellow progeny, what proportion is wrinkled?